Key

Math 4 Name ______________________

5-10 Chain Rule Date______

You are now a master of the product rule and quotient rule. I'm sure you've come to love them like family. You have used the power rule to derive functions such as $f(x) = (5x+2)^2$ by writing this as two functions and applying the product rule. But what if you had something like this: $q(x) = (5x+2)^{73}$? The product rule won't be much help for that. This is where the **chain rule** steps in. One last time, take a look at the examples below and try to come up with the chain rule from those.

Example 1:

$$f(x) = (5x+2)^{73}$$

$$f'(x) = 73(5x+2)^{72} \cdot (5)$$

Example 2: $y = (3x^2 + 7x - 11)^6$

$$\frac{dy}{dx} = 6(3x^2 + 7x - 11)^5 \cdot (6x+7)$$

Chain Rule: If $y = [g(x)]^n$, then $\frac{dy}{dx} = \boxed{n} \cdot \boxed{[g(x)]^{n-1}} \cdot \boxed{g'(x)}$

Example 3: $y = (2x^2 + 1)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(2x^2+1)^{-\frac{1}{2}}(4x)$$

$$= 2x(2x^2+1)^{-\frac{1}{2}}$$

$$= \boxed{\frac{2x}{\sqrt{2x^2+1}}}$$

Example 4: $y = (6x^3 + 2x^2 - 4x + 1)^3$

$$y' = 3(6x^3+2x^2-4x+1)^2(18x^2+4x-4)$$

$$= \boxed{(54x^2+12x-12)(6x^3+2x^2-4x+1)^2}$$

Practice! For each function, find the first derivative.

1. $f(x) = x^3$

$\boxed{f'(x) = 3x^2}$

No chain rule needed!

2. $f(x) = (-5x+6)^4$

$f'(x) = 4(-5x+6)^3(-5)$

$= \boxed{-20(-5x+6)^3}$

3. $y = (2x^2-5)^{10}$

$\frac{dy}{dx} = 10(2x^2-5)^9(4x)$

$= \boxed{40x(2x^2-5)^9}$

4. $c(x) = \sqrt{4x+7} = (4x+7)^{\frac{1}{2}}$

$c'(x) = \frac{1}{2}(4x+7)^{-\frac{1}{2}} \cdot 4$

$= \boxed{\frac{2}{\sqrt{4x+7}}}$

5. $t(x) = (x^2-3)^1$

$\boxed{t'(x) = 2x}$

No chain rule needed!

6. $q(x) = (7x-3)^{-4}$

$q'(x) = -4(7x-3)^{-5}(7)$

$= -28(7x-3)^{-5}$

$= \boxed{\frac{-28}{(7x-3)^5}}$

7. $y = \frac{1}{(9x+13)^2} = (9x+13)^{-2}$

$\frac{dy}{dx} = -2(9x+13)^{-3}(9)$

$= -18(9x+13)^{-3}$

$= \boxed{\frac{-18}{(9x+13)^3}}$

8. $g(x) = (2x^2+3x-2)^2$

$g'(x) = 2(2x^2+3x-2)(4x+3)$

$= \boxed{(8x+6)(2x^2+3x-2)}$

Can expand this if you'd like.

9. $y = (2x-1)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(2x-1)^{-\frac{1}{2}}(2)$

$= (2x-1)^{-\frac{1}{2}}$

$= \boxed{\frac{1}{\sqrt{2x-1}}}$

10. $m(x) = (16-3x^4)^{-\frac{2}{3}}$

$m'(x) = -\frac{2}{3}(16-3x^4)^{-\frac{5}{3}}(-12x^3)$

$= 8x^3(16-3x^4)^{-\frac{5}{3}}$

$= \boxed{\frac{8x^3}{\sqrt[5]{(16-3x^4)^3}}}$